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NEW ANALYTICAL APROACH FOR COSMIC RAY IONIZATION MODELING IN PLANETARY ENVIRONMENTS BY USING THE IONIZATION YIELD FUNCTIONS

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Abstract

The cosmic rays (CR) influence the ionization and electric parameters in the atmosphere and also the chemical processes (ozone creation and depletion in the stratosphere) in it. CR ionize the whole middle and lower atmosphere, i.e. the strato-mesosphere and troposphere. The cosmic rays are a key factor affecting the atmospheric chemistry and the space weather and space climate in the Earth's environment.

Two approaches have been developed to compute the CR ionization, for example:
1) the analytical model CORIMIA - COsmic Ray Ionization Model for Ionosphere and Atmosphere, and
2) the statistical CORSIKA code, including the FLUKA Monte Carlo package, which is based on a Monte Carlo simulation of the atmospheric cascade. Usually the analytical models use the ionizing capability C function, while the statistical simulations utilize the ionization yield Y function.

In the present work we find connection between the functions C and Y and we will proceed to a unified approach to the calculation of atmospheric ionization due to cosmic rays with galactic (GCR), solar (SCR) and interplanetary (anomalous CR, ACR) origin. Formulas for ionizing capability and ionization yield functions are derived for general case, for relativistic approximation (GCR) and for sub-relativistic case (SCR and ACR). The input parameters in the proposed model include the full composition of the CR nuclei groups as follows: protons (p, $Z = 1$), alpha particles (α, $Z = 2$), and HEZ particles: Light (L, $3 \le Z \le 5$), Medium (M, $6 \le Z \le 9$), Heavy (H, $10 \le Z \le 19$), Very Heavy (VH, $Z \ge 20$) and Super Heavy (SH, $Z \ge 30$) groups of nuclei.

Some practical applications of the obtained results are discussed. The results of the full Monte Carlo simulation which are tabulated in a form of the ionization yield function can be applied much more widely. With the help of the achievements of this work the analytical model CORIMIA can use the results of statistical code CORSIKA that will *allow comparison between the two approaches. As is known by CORSIKA is possible to estimate the CR ionization below 20-25 km and by the model CORIMIA - above this altitude. So those two approaches are complementary.*

The proposed approach can be used at the quantitative consideration and analysis of the solar-terrestrial relationships and the problems of space weather and space climate. It is a theoretical approach and consequently can be applied also for the calculation of ionization effects in the planetary ionospheres and atmospheres.

Introduction

Presently, there are numerous arguments suggesting that the solar activity variability affects the global climate in different aspects and on different timescales. Several possible mechanisms have been suggested, which can be responsible for the observed relation between the solar variability and the climate: via the changing solar irradiance; by the UV heating of the stratosphere and the consequent circulation variations; or by cosmic rays affecting the cloud formation [1].

Galactic cosmic rays (GCR), solar cosmic rays (SCR) and anomalous cosmic rays (ACR) are responsible for the ionization in the atmosphere and lower ionosphere by electromagnetic and nuclear interactions [2, 3]. Because of their high energy (up to 10^{21}eV) the particles, which originate from the Galaxy can react and produce ionization in the atmosphere until being absorbed in it. The influence of cosmic rays (CR) is important for the electric parameters and the chemical processes - ozone creation and depletion in the stratosphere [2, 4]. Therefore the cosmic rays determine the electric conductivity and influence the global electric circuit of the Earth. There is also combined hypothesis of CR-UV impact on the solar-atmosphere relationships [1].

Effectively the cosmic rays ionize the whole middle and lower atmosphere, i.e. the strato-mesosphere and troposphere. GCR create an independent cosmic ray layer in the lower part (50-80 km) of the ionospheric *D*-region [5, 6, 7, 8]. This cosmic ray layer was called *CR*- or *C*-layer. The contribution (up to several hundred electrons per cubic centimeter) of cosmic rays in the formation of *C*-layer has been confirmed experimentally by the data of rocket flights [9] and absorption measurements by the ionosphere’s propagation of long radio waves [7, 8]. Initially the ionization, i.e. the electron production rate q [$cm^{-3}s^{-1}$], of the galactic cosmic rays in the atmosphere was determined by the empirical formula given in [9]. The electron production rate *q(h)* in the ionosphere have been theoretically calculated as a function of height *h* firstly in the work [5, 6]. The results obtained have been generalized in [10, 11]. There is used a realistic curved atmosphere to allow computing CR electron production rate profiles. The chemical composition of the atmosphere was taken as N_2, O_2 and Ar in the volume fractions of 78.1%, 21% and 0.9%, respectively. The ionosphere's electron density profiles were modeled by four latitudes: 0^o, 30^o, 41^o and 55^o.

Two approaches have been developed to compute the cosmic ray ionization: analytical and statistical. The mentioned works [10, 11] and the improved model CORIMIA - *COsmic Ray Ionization Model for Ionosphere and Atmosphere* (see the full description in [12]) are created by an analytical approximation of the ionization losses function of Bohr-Bethe-Bloch, while the statistical models are based on a Monte Carlo simulation of the atmospheric cascade [1, 13, 14]. The latter method is an improved Monte Carlo model which is based on an updated version of the CORSIKA code, including the FLUKA Monte Carlo package to simulate the low-energy nuclear interactions, and explicitly the direct ionization by primary CR particles [1, 13, 14].

Usually the analytical models [12] use the ionizing capability *C* function [10, 11], while the statistical Monte Carlo simulations utilize the ionization yield *Y* function [1, 13, 14]. In the present work we will find connections between the functions *C* and *Y* and we will proceed to a unified approach to the calculation of atmospheric CR ionization.

In this work also formulas for ionizing capability and ionization yield functions will be derived for relativistic (GCR) approximation, i.e. the particles with relative velocity

$$\beta = v/c \approx 1$$

where *v* is velocity of the CR nuclei and *c* is the light velocity. Then the sub-relativistic (SCR and ACR) case

$$\beta = v/c < 1$$

will be studied detailed and thus the results will be essentially generalized.

1. Model for Electron Production Rate Profiles

During their penetration in the atmosphere the cosmic rays cause ionization of the neutral components. Therefore free electrons are emitted. The ionization production rate q [$cm^{-3}s^{-1}$] at height h [km] is calculated as a superposition of the effect of different groups of nuclei in the composition of cosmic rays. A three dimensional model for the electron production rate is used [5, 6, 10, 11, 15]:

(1)

$$q(h) = \sum_i q_i(h) = \sum_i \frac{1}{Q} \int_{E_i}^{\infty} \int_{\alpha=0}^{2\pi} \int_{\theta=1}^{\pi/2+\Delta\theta} D_i(E) \left(\frac{dE}{dh} \right)_i \sin(\theta)\, d\theta\, d\alpha\, dE,$$

where Q = 35 eV is energy for formation of one electron-ion pair. α is the azimuth angle, θ is the angle towards the vertical, $\Delta\theta$ takes into account that at a given height the particles can penetrate from the space angle (0^o, θ_{max} = = 90 + $\Delta\theta$), which is greater than the upper hemisphere angle (0^o, 90^o) for a flat model. E_i are the corresponding energy cut-offs. The summation of the ionization integral (1) is made on the all groups of nuclei: protons p (i = 1), alpha particles α (i = 2) and heavier groups of nuclei - HZE nuclei: light L (i = 3), medium M (i = 4), heavy H (i = 5), very heavy VH (i = 6) and super heavy SH (i = 7) in the CR composition.

In the expression (1) E is the full energy (GeV/nucl) of the penetrating particles and the differential spectrum [$cm^{-2}\ s^{-1}sr^{-1}GeV^{-1}$] is in the form:

(2) $D(E) = K\, E^{-\gamma}$

K and γ are constants for the galactic cosmic rays; (dE/dh) are the ionization losses of CR particles according to the formula of Bohr-Bethe-Bloch [9].

Currently, there are mainly three types of models for calculation of the ionization rates in the ionosphere and atmosphere [16]:

1) thin target model (for the ionosphere, above 50 km),

2) intermediate target model (for the ozonosphere, 30-50 km), and

3) full target model (for the troposphere and lower stratosphere, below 25-30 km).

The present paper concerns: 1) the thin target, and 2) the intermediate target models. For the lower and middle atmosphere below 25-30 km, the full target models 3): CORSIKA, GEANT4, etc., are used [16]. These models are not applicable for altitudes above 25-30 km because of the small atmosphere depths there. They require atmospheric depths of at least 3-4 g/cm^2.

Since the galactic cosmic rays penetrate isotropically from the upper hemisphere from (1) it follows the more simple expression:

$$(3) \qquad q(h) = \frac{2\pi}{Q} \int_{E_C}^{\infty} D(E) \left(\frac{dE}{dh} \right) dE$$

as the multiplier 2π shows this isotropic influx of the galactic particles; E_C is the corresponding geomagnetic cutoff energy in GeV/nucl.

The variable of integral (3) can be not only the energy of the CR particles, but also their rigidity R (GV). In this case the differential spectrum [cm^{-2} s^{-1}sr^{-1}GV^{-1}] will be

$$(4) \qquad D(R) = K\, R^{-\gamma}$$

as the energy cutoff E_{i} in (3) must be replaced by the corresponding geomagnetic threshold of rigidity R_{c} (GV).

1.1. Analytical and Statistical Approaches in Ionization Models and Relationships between them

At *the analytical approach* the electron production rate $q(h)$ is closely connected with the cosmic ray ionizing capability. In general the ionizing capability function $C(h)$ at given altitude h with atmospheric density ρ [g cm^{-3}] is determined by the expression [10, 11]:

$$(5) \qquad C(h) = q(h) / \rho(h)$$

This function physically represents the number of electron-ion pairs produced in one gram of matter (in this case atmospheric air) per second and characterizes the ionization effectiveness of the radiation factor.

In *the statistical approach* is calculated initially the ionization yield function Y [1, 14, 16] which gives the number of ion pairs produced in one gram of the ambient air at a given atmospheric depth by one nucleon of the primary cosmic ray particle with the given energy per nucleon. Then, the integration by energy is carried out. In this approach all results depend on exact atmospheric density profile. In order to minimize this uncertainty, the CR ionization is computed per gram of the atmospheric matter rather than per cm^3. In this case, the uncertainties of the *q(h)*, computed using different atmospheric density profiles, do not exceed 1–2% in the low troposphere, which is less than statistical errors of computations.

We want to combine the mentioned two approaches. For this purpose we introduce the function *Y* [1, 13, 14, 16] in the expression (3) after which we derive the formula:

(6) $$q(h) = \int_{E_C}^{\infty} D(E) Y(E,h) \rho(h) dE$$

Here $\rho(h)$ is the density of atmosphere [g cm^{-3}] at height *h* and the ionization yield function *Y* [electron-ion pairs sr cm^2 g^{-1}] can be represented as:

(7) $$Y(E,h) = \frac{2\pi}{Q}\left(\frac{1}{\rho}\frac{dE}{dh}\right)$$

Thus, the wanted connection between the functions *C*(*h*) and *Y*(*h*) will be:

(8) $$C(>E_C) = \int_{E_C}^{\infty} D(E) Y(E,h) dE$$

Actually the CR ionizing capability *C*(*h*) depends on the function of ionization losses and ionization potential of the medium *Q*, on the type of CR nuclei (or group of nuclei: p, He, L, M, H, VH and SH), on their spectrum and geomagnetic threshold of rigidity.

1.2. General Formulas for Ionization Yield Function

Using the relation (7) and the expression for ionization losses [5, 6, 10, 11] we can write the following formula for the ionization yield function:

(9) $$Y(E) = \frac{2\pi}{Q}\frac{0.3\,Z^2}{\beta^2}\left(\ln\frac{E}{E_0}\beta + 4.5 - \frac{\beta^2}{2}\right)$$

where Z is the charge of the penetrating CR particle,
$\beta = v/c$ is the relative velocity, where v is velocity of the particle and c is the light velocity;
E and E_0 are the full energy and rest energy of the CR nuclei.

The relationship between E and E_0 is established using the relativistic equations [11]:

$$(10) \qquad E = \frac{E_0}{\sqrt{1-\beta^2}} \quad \text{or} \quad \beta = \frac{\sqrt{E^2 - E_0^2}}{E}$$

$$(11) \qquad E = Mc^2 \quad \text{and} \quad E_0 = M_0 c^2$$

where M and M_0 are the mass and rest mass of the particles. If we substitute (10) in (9) we obtain the following expression for the ionization yield function of penetrating CR particles:

$$(12) \qquad Y(E) = \frac{2\pi}{Q} 0.3\, Z^2 \frac{E^2}{E^2 - E_0^2}\left(\ln\sqrt{E^2 - E_0^2} + 4.5 - \frac{1}{2}\frac{E^2 - E_0^2}{E^2} \right)$$

The rest mass of a proton is 938 MeV c^{-2}, i.e. $E_0 = 0.938$ GeV c^{-2}. If we put

$$(13) \qquad E_0^2 = 0.879844 \approx 0.88$$

in (12) we receive the following more concrete presentation:

$$(14) \qquad Y(E) = \frac{2\pi}{Q} 0.3\, Z^2 \frac{E^2}{E^2 - 0.88}\left(\ln\sqrt{E^2 - 0.88} + 4.5 - \frac{1}{2}\frac{E^2 - 0.88}{E^2} \right)$$

In the development of some problems it is more convenient to use simplified and symmetric equations. That is why sometimes the energy in cosmic ray physics is expressed in units $E_0 = 0.938$ GeV. In this case, the equation (14) will pass into:

$$(15) \qquad Y(E) = \frac{2\pi}{Q} 0.3\, Z^2 \frac{E^2}{E^2 - 1}\left(\ln\sqrt{E^2 - 1} + 4.5 - \frac{1}{2}\frac{E^2 - 1}{E^2} \right)$$

This expression will be exploited in the future when examining the variety of cases of cosmic ray influences of the ionization state of the middle atmosphere.

2. Relativistic Case – Galactic Cosmic Rays

2.1. Relativistic Approximation

Since the galactic cosmic rays are with relativistic energies, then the condition

$$\beta = v / c \approx 1$$

(exactly fulfilled at lower and middle geomagnetic latitudes) is valid. After assuming this ratio from ionization yield function equation (9) follows:

(16) $$Y(E) = \frac{2\pi}{Q} 0.3\, Z^2 \left(\ln \frac{E}{E_0} + 4 \right)$$

This means that

(17) $E >> E_0$ or $E \approx E_k$

where

(18) $E_k = E - E_0$

is the kinetic energy of the particle. If we place ln E_0 = ln 0.938 in (16), we will receive the formula:

(19) $$Y(E) = \frac{2\pi}{Q} 0.3\, Z^2 (\ln E + 4.064)$$

or

(20) $$Y(E) = \frac{2\pi}{Q} 0.3\, Z^2 (\ln E + 4)$$

if the energy is expressed in units E_0 = 0.938 GeV.

2.2. Formulas for Ionizing Capability

After substitution the last derived expression (20) in (8) and after the appropriate solution of the integral, we will receive the following formula for the ionizing capability:

(21) $$C(>E_C) = 5.4\times10^4 D(>E_C)\,Z^2\left(\ln E_C + \frac{1}{\gamma-1} + 4\right)$$

where $D(>E_c)$ is the CR integral spectrum and γ = 2,5 - 2,6 is the exponent in differential spectrum [5, 6, 11]. If this exponent γ laid on its average value and if we take into account the relationship $\ln E_c = 2{,}3\lg E_c$, we will receive the more convenient formula:

(22) $$C(>E_C) = 1.24\times10^5 D(>E_C)(\lg E_C + 2)$$

as the energy is expressed everywhere in a natural unit E_0 = 0.938 GeV.

By the formulas (20, 21) can be found another connection between the integrated ionizing capability and the ionization yield function:

(23) $$C(>E_C) = D(>E_C)\left(Y(E_C) + 5.4\times10^4\frac{Z^2}{\gamma-1}\right)$$

Y function may be represented also through geomagnetic rigidity *R* taking into account the ratio [3]:

(24) $R = (A/Z)\,E$

A is the atomic weight and *Z* is the charge of the particle. If we use the relationship (24), the following expression in the relativistic case is obtained from (21):

(25) $$C(>R_C) = 5.4\times10^4 D(>R_C)Z^2\left(\ln\frac{Z}{A}R_C + \frac{1}{\gamma-1} + 4\right)$$

In this formula may be accounted the different characteristics of cosmic ray particles, for example relations $A/Z = 1$ for protons and $A/Z \approx 2$ for heavier nuclei. Thus this formula can be further elaborated.

It should be noted that the particles with the ratio of $A/Z > 1$ are less deflected by the geomagnetic field, which, in combination with their weaker heliospheric modulation, makes them very important for the CR ionization [1].

If we take into account the real GCR composition, we can deduce the following a simplified expression:

(26) $$C(>R_C) = 1.56\times10^5 D(>R_C)(\ln R_C + 4.15)$$

As the galactic cosmic rays consist of 87% protons (from all nuclei), it is convenient the ionizing capability to be expressed by the integral spectrum of protons:

(27) $$C(>R_C) = 1.8\times10^5 D_p(>R_C)(\ln R_C + 4.15)$$

which is suitable for practical calculations.

The following two factors significantly affect ionizing capability C:

i) latitudinal effect, and ii) 11-years variations of galactic cosmic rays.

i) The latitudinal effect can be shown clearly by expressing the integral spectrum and geomagnetic cut-off rigidity in (26) trough the rigidity and then trough the geomagnetic latitude λ_m. For this purpose we replace the spectrum (4) in (26) , as a result of which we obtain:

(28) $$C(>R_C) = 1.8\times10^5 K_p \frac{\ln R_C + 4.15}{(\gamma-1)R_C^{(\gamma-1)}}$$

Now we will express R_c by means of the approximation [10, 11]

(28) $$R_C \cong 14.9\cos^4\lambda_m$$

in which the geomagnetic field is represented as a dipole. It follows from (28) at

$$\gamma = 2.5$$

that

(29) $$C(>R_C) = 6.3\times10^3 K_p \frac{0.576\ln\cos\lambda_m + 1}{\cos\lambda_m^6}$$

which visibly expresses the latitudinal effect in the distribution of cosmic ray ionizing capability.

ii) The 11-years variations of primary galactic cosmic rays. Their ionizing capability is expressed in the opposite variations of the variations of solar activity levels and the cosmic ray flux. At polar geomagnetic latitudes the integral particle flux increases during the period of the solar minimum twice in relation to the flux in solar maximum, while at equatorial region the variation is small. At middle latitudes the variation is significant.

The galactic cosmic rays initiate a nucleonic-electromagnetic cascade in the atmosphere, affecting its physical-chemical properties and ion balance [1, 13, 14, 16]. This is a dominant source of ionization of the troposphere and stratosphere [1, 11, 17]. Therefore a detailed model of the GCR ionization makes a solid basis for an investigation of the mechanisms of solar-terrestrial physics. The galactic cosmic rays are a key factor affecting the atmospheric chemistry (e.g. the ozone distribution [17, 18]) and the space weather in the extraterrestrial space.

3. Sub-relativistic Case – Solar and Anomalous Cosmic Rays

All types of cosmic rays (GCR, SCR and ACR) influence in a different manner the iono/atmosphere systems of the Earth and planets. The solar cosmic rays produced in solar flares (and in some other high-energy solar processes) are one of most important manifestation of solar activity and one of the main agents in solar-terrestrial relationships. The solar cosmic rays have sufficient energy and intensity to raise radiation levels on Earth's surface. This event is termed a "Ground Level Enhancement" (GLE) [19]. Solar cosmic rays consist of protons, electrons, helium ions, and HZE ions with energy ranging from a few tens of keV to GeV. They are ejected primarily in solar flares and coronal mass ejections (CME). They have a composition similar to that of the Sun, and are produced in the corona by shock acceleration, or when part of the solar magnetic field reconfigures itself.

Moreover the anomalous cosmic rays (ACR) are accelerated in the outer heliosphere from pick-up ions that primarily originate as interstellar neutrals. ACR, among the most energetic particle radiation in the Solar system, are thought to be produced at the termination shock - the boundary at the edge of the Solar system where the solar wind abruptly slows.

Anomalous cosmic rays include large quantities of helium, oxygen, neon, and other elements with high ionization potentials, that is, they require a great deal of energy to ionize, or form ions [20]. ACR are a tool for studying the movement of energetic particles within the Solar system, for learning the general properties of the heliosphere, and for studying the nature of interstellar material itself. ACR are thought to represent a sample of the very local interstellar medium. They are not thought to have experienced such violent processes as GCR and SCR, and they have a lower speed and energy.

The purpose of the present work is to derive new formulas for ionizing capability *C* and ionization yield functions *Y* in the case of the solar cosmic rays and anomalous cosmic rays, which requires the obtaining of new convenient sub-relativistic approximations according to their energy.

3.1. Formulas for Ionization Losses Function

The ionization losses function of Bohr-Bethe-Bloch is the basis for calculating of electron-ion production rate profiles (1, 3) and ionizing capability at height *h* [11]:

$$(30)\quad -\frac{dE}{dh} = 0.3\rho(h)Z^2 \frac{E^2}{E^2 - E_0^2}\left[\ln\sqrt{E^2 - E_0^2} + 4.5 - \frac{1}{2}\frac{E^2 - E_0^2}{E^2}\right]$$

where E_0 is the rest mass of the particles and $\rho(h)$ is the density [g cm-3] of atmosphere at height *h*. The rest mass of the proton is 938 MeV c^{-2}, i.e. E_0 = = 0.938 GeV c^{-2}. If we put

$$E_0^2 = 0.879844 \approx 0.88$$

in (30) we receive the following more concrete presentation:

$$(31)\quad -\frac{dE}{dh} = 0.3\rho(h)Z^2 \frac{E^2}{E^2 - 0.88}\left[\ln\sqrt{E^2 - 0.88} + 4.5 - \frac{1}{2}\frac{E^2 - 0.88}{E^2}\right]$$

In the development of some problems it is more convenient to use simplified and symmetric equations. That is why sometimes the energy in cosmic ray physics is expressed in units E_0 = 0.938 GeV. In this case, the equation (31) will pass into:

(32) $$-\frac{dE}{dh} = 0.3\rho(h)Z^2 \frac{E^2}{E^2-1}\left[\ln\sqrt{E^2-1} + 4.5 - \frac{1}{2}\frac{E^2-1}{E^2}\right]$$

We will use this expression most often.

3.2. Complementary Expressions for Ionization Yield Function

The ionization yield function *Y* [electron-ion pairs sr cm^2 g^{-1}] is used usually in the statistical Monte Carlo simulations [1, 3, 13, 14]. It can be represented as (7)

$$Y(E,h) = \frac{2\pi}{Q}\left(\frac{1}{\rho}\frac{dE}{dh}\right)$$

Taking into account formulas (30, 31, 32), we can write from here similar expressions for ionization yield function in different representations (using the full energy *E* or kinetic energy E_k) according to specific task:

(33) $$Y(E) = 5.4\times10^4 Z^2 \frac{E^2}{E^2-E_0^2}\left(\ln\sqrt{E^2-E_0^2} + 4.5 - \frac{1}{2}\frac{E^2-E_0^2}{E^2}\right)$$

(34)
$$Y(E) = 5.4\times10^4 Z^2 \frac{(E_0+E_k)^2}{E_k(2E_0+E_k)}\left(\ln\sqrt{E_k(2E_0+E_k)} + 4.5 - \frac{1}{2}\frac{E_k(2E_0+E_k)}{(E_0+E_k)^2}\right)$$

(35)
$$Y(E) = 5.4\times10^4\ Z^2 \frac{(0.938+E_k)^2}{E_k(1.876+E_k)}\left(\ln\sqrt{E_k(1.876+E_k)} + 4.5 - \frac{1}{2}\frac{E_k(1.876+E_k)}{(0.938+E_k)^2}\right)$$

Here is used the following relationship between the total energy *E*, the rest energy E_0 and the kinetic energy E_k:

(36) $$E = E_0 + E_k$$

If we express the energy in units E_0 = 0.938 GeV, i.e. $E = 1 + E_k$ we will receive the frequently used and convenient formula

$$(37)\qquad Y(E) = 5.4\times 10^4\ Z^2 \frac{(1+E_k)^2}{E_k(2+E_k)}\left(\ln\sqrt{E_k(2+E_k)} + 4.5 - \frac{1}{2}\frac{E_k(2+E_k)}{(1+E_k)^2}\right)$$

3.3. Sub-relativistic Approximation

In the previous part 2. was studied the relativistic case

$$(38)\qquad \beta = v/c \approx 1$$

This concerns the galactic CR at middle and lower latitudes. However, at higher geomagnetic latitudes the galactic CR penetrate with

$$(39)\qquad \beta = v/c < 1$$

In a similar way during geomagnetic storms the geomagnetic threshold of rigidity Rc decreases and CR particles can penetrate with velocities smaller than the light velocity c, i.e. they are sub-relativistic. These CR nuclei produce bigger ionization. Therefore, here we will examine the more general case of sub-relativistic ionization. For this purpose we will present the energy *E* through rigidity *R* using the formula [10, 11]:

$$(40)\qquad E = 1 + E_\mathrm{k} = \left[1 + \left(\frac{Z}{A}R^2\right)^{1/2}\right]$$

in the expression for ionization losses (30, 31, 32). We also use the sub-relativistic presentation for the relative velocity [2, 18]:

$$(41)\qquad \beta = \frac{R}{\left[R^2 + (A/Z)^2\right]^{1/2}}$$

Here *Z* is the charge and *A* atomic weight of the penetrating CR particles.

Substituting (40) and (41) in the expression (32) we can obtain the following formula for the ionization losses:

$$-\left(\frac{dR}{dh}\right) = 0.3\rho(h)Z^2\left[1+\left(\frac{A}{ZR}\right)^2\right]\left[\ln\frac{Z}{A}R + 4.5 - 0.5\frac{1}{1+(A/ZR)^2}\right] \quad (42)$$

3.4. Ionizing Capability of Sub-relativistic Particles

By replacing the last expression (42) and the spectrum (4) in (3), the solution of the integral for $q(h)$ and taking into account the definition (5):

$C(h) = q(h) / \rho(h)$

we will receive the following formula for the ionizing capability:

$$C(h) = 5.4\times10^4 \sum_{\mathrm{i}=1}^{7} D_i(> R_c) Z_\mathrm{i}^2 \left(\ln\frac{Z_\mathrm{i}}{A_\mathrm{i}}R_c + 4.5\right)\left[1+\frac{\gamma-1}{\gamma+1}\left(\frac{A_\mathrm{i}}{Z_\mathrm{i}R_c}\right)^2\right] \quad (43)$$

where $D(> R_c)$ are the CR integral spectrums of different types of nuclei [9, 11].

The same result is obtained after replacement of the ionization yield function Y (33-35, 37) in the general expression (3) which in this case will be in the equivalent form (6):

$$q(h) = \int_{E_C}^{\infty} D(E) Y(E,h) \rho(h) dE$$

or

$$q(h) = \int_{R_C}^{\infty} D(R) Y(R,h) \rho(h) dR \quad (44)$$

In the formula (43) we can notice a group of terms corresponding to the relativistic CR particles [11]. If we indicate q of relativistic CR with $q(\beta = 1)$ and q of sub-relativistic ones with $q(\beta < 1)$, then from (43) we can write

$$C(\beta<1)\approx\sum_{i=1}^{7}C_i(\beta=1)\left[1+\frac{\gamma-1}{\gamma+1}\left(\frac{A_i}{Z_i R}\right)^2\right]$$

(45) $$=\left(1+\frac{0.43}{R^2}\right)C_p(\beta=1)+\left(1+\frac{1.71}{R^2}\right)\sum_{i=2}^{7}C_i(\beta=1)$$

$$=\left(1+\frac{1.35}{R^2}\right)C(\beta=1)$$

where C_p is the ionizing capability of the protons and C_i - of heavier nuclei: Helium, L, M, H, VH and SH groups of nuclei in the composition of the primary cosmic rays. In equation (45) the capabilities of all particles in the galactic CR composition are summed up. We also took into account that Z_i / A_i = 1 for protons and Z_i / A_i ≈ 1/2 for heavier ($Z_i \geq 2$) nuclei. In the expressions (43, 45) must be replaced the real spectrum and chemical composition of the galactic cosmic rays.

It is clear that at $R < 2$ GV the expressions (42, 43, 45) generalize the results in the part **2. Relativistic Case - GCR** in the present paper. In this way we can write the relationship

(46) $C(\beta < 1) = C(\beta=1)\ (1 + p)$

as the factor p takes into account the sub-relativistic contribution. At rigidity $R > 2$-3 GV the two groups of formulas practically coincide.

Analogously of the expression (46) can be derived the following formula for the ionization yield function Y:

(47) $Y(\beta < 1) = Y(\beta = 1)\ (1 + p)$

This formula will be considered and analyzed in detail in a future study.

3.5. Spectra and Ionization Yield Functions of Sub-relativistic Particles

The spectra of solar cosmic rays are represented before all in the form of spectrum by kinetic energy [9, 19]:

(48) $D(E_k) = K\, E_k^{-n}$

The experimental data show that the parameter n has values $n \sim 3 - 7$ depending on the rigidity of the solar particles and the acceleration mechanisms on the Sun [19]. For example the spectrum of solar CR during the greatest GLE, observed on February 23, 1956 is [10, 21]:

(49) $D(E_k) = 4 \times 3310^9\ E_k^{-5}$ proton/cm^2 sec ster MeV

This is the extreme case. The shapes and relative slopes of the spectra of different solar energetic particles: protons, alphas and electrons, may vary among cases. There is significant variation from event to event.

We will use the results obtained in the previous parts. Due to the type of the spectrum (48, 49) the ionization yield function may be expressed by means of the kinetic energy E_k. For this purpose we determine the rigidity R of the solar particles with charge Z and atomic weight A [11]

(50) $$\frac{Z}{A}R = \left(E^2 - 1\right)^{\frac{1}{2}} = \left[E_k\left(2 + E_k\right)\right]^{\frac{1}{2}}$$

which replaced in equation (9) gives the following expression:

(51) $$Y(R) = 5.4 \times 10^{-2} Z^2 \left[1 + \frac{1}{E_k\left(2 + E_k\right)}\right] \times$$
$$\times \left[\ln\left[E_k\left(2 + E_k\right)\right]^{\frac{1}{2}} + 4.65 - \frac{1}{2}\left(1 - \frac{1}{\left(1 + E_k\right)^2}\right)\right]$$

where E is the full energy (GeV) of the penetrating particles and E_k their kinetic energy in GeV.

In their penetration in the ionosphere and atmosphere the particles decrease their energy E_k which leads to an increase of the ionization yield function. So that the kinetic energy as well as the ionization yield function themselves will depend on the altitude. Now we shall make some considerations concerning the determination of the law of energy decrease $E_k(h)$. In order to obtain this law the ionization yield function (5) can be simplified for solar cosmic rays, which are in the majority cases sub-relativistic ($E_k < 500 - 600$ MeV). Actually for these energies must be taken into account the absorption of solar particles in the iono/atmosphere. Besides for energies E_k < 500 – 600 MeV the solar cosmic rays have the bigest intensity and ionization yield function Y and therefore a maximal iono/atmospheric influence.

Using the fact that for the solar CR particles the inequality E_k < 2 GeV is fulfilled, the following simplified formula is derived from the complete expression (51):

$$(52)\qquad Y(E_k) = 1.55 \times 10^{-2} Z^2 \frac{\ln E_k + 10}{E_k}$$

If in (52) we ignore the variation of the logarithmic term (it changes much more slowly than the term E_k^{-1}), and we solve the equation for ionization path (g cm^{-2}) of the particles [11], we will receive the law that is sought:

$$(53)\qquad E_k(h) = \left(E_k^2 - 10^3 \tilde{h} Z^2 \sec\theta\right)^{\frac{1}{2}}$$

Here the energy is expressed in MeV/nucleon; $\tilde{h}$ is the atmospheric depth (g cm^{-2}); θ is the penetration angle of solar CR particles in the iono/atmosphere.

3.6. Ionizing Capability of Solar and Anomalous Cosmic Rays

C function (5) physically represents the number of electron-ion pairs produced in one gram of matter (in this case atmospheric air) per second and characterizes the ionization effectiveness of the radiation factor [10, 11]. Taking into account the law of energy E_k decrease (53) and using (51) for the general expression of ionizing capability we can write the following general expression:

$$(54)\qquad C(h) = 1.8 \times 10^5 \sum_{i=1}^{n} \int_{E_{k\max}}^{\infty} \int_0^{90^0} D_i(E_k) 0.3 Z^2 \times$$

$$\times \frac{\left[\left(E_k^2 - 10^3 \tilde{h} Z_i^2 \sec\theta\right)^{\frac{1}{2}} + 1\right]^2}{\left(E_k^2 - 10^3 \tilde{h} Z_i^2 \sec\theta\right) + 2\left(E_k^2 - 10^3 \tilde{h} Z_i^2 \sec\theta\right)^{\frac{1}{2}}}$$

$$\times\left\{0.5\ln\left(E_{\mathrm{k}}^2-10^3\tilde{h}Z_{\mathrm{i}}^2\sec\theta\right)^{\frac{1}{2}}\left[\left(E_{\mathrm{k}}^2-10^3\tilde{h}Z_{\mathrm{i}}^2\sec\theta\right)^{\frac{1}{2}}+2\right]+4.65\right.$$

$$\left.-0.5\frac{\left(E_{\mathrm{k}}^2-10^3\tilde{h}Z_{\mathrm{i}}^2\sec\theta\right)^{\frac{1}{2}}\left[2+\left(E_{\mathrm{k}}^2-10^3\tilde{h}Z_{\mathrm{i}}^2\sec\theta\right)^{\frac{1}{2}}\right]}{\left[\left(E_{\mathrm{k}}^2-10^3\tilde{h}Z_{\mathrm{i}}^2\sec\theta\right)^{\frac{1}{2}}+1\right]^2}\right\}\sin\theta\,d\theta\,dE_{\mathrm{k}}$$

The summation is carried out by all nuclei in the composition of solar cosmic rays: protons, helium (alphas) etc. The boundary of the integral E_{kmax} is determined by the following manner:

$$(55)\qquad E_{\mathrm{k\,max}}=\max\begin{cases}E_{\mathrm{Ri}}=10^3\left\{\left[\left(\dfrac{Z_{\mathrm{i}}}{A_{\mathrm{i}}}\right)^2R^2+1\right]^{\frac{1}{2}}-1\right\}\\ E_{\mathrm{Ai}}=\left(10^3\tilde{h}Z^2\sec\theta+\varepsilon\right)^{\frac{1}{2}}\\ E_{\mathrm{Ei}}\end{cases}$$

i.e. for each altitude E_{kmax} is the biggest of the three energies: the energy corresponding to geomagnetic cutoff rigidity E_{Ri}, to the atmospheric cutoff E_{Ai}, and to the electric energy cutoff E_{Ei} outside the geomagnetic field. The energy ε is about 0.1 MeV [11].

3.7. Lower Energy Approximation

The general formula (54) is complicated because of many not significant terms. If these terms are ignored, as it happened in equation (52), a simplified formula can be derived from (54) for the ionizing capability of solar cosmic rays:

$$(56)\qquad C(h) = 5.4\times 10^4 \sum_{i=1}^{n} \int_{E_{\mathrm{k\,max}}}^{\infty} \int_{0}^{90^\circ} D_i(E_k) 83 Z_i^2$$

$$\times \frac{\ln\left(E_k^2 - 10^3 \tilde{h} Z_i^2 \sec\theta\right)^{\frac{1}{2}} + 3.2}{\left(E_k^2 - 10^3 \tilde{h} Z_i^2 \sec\theta\right)^{\frac{1}{2}}} \sin\theta \, d\theta \, dE_k$$

as the integral boundary E_{kmax} is the same as (55). This formula is much more convenient than the full expression (54), as the difference in the calculations was below 5%. For comparison, the experimental error in the rocket flight measurements is approximately in this range.

3.8. Generalisation of the Results for Spherical Iono/Atmosphere

The previous formulas refer to flat model of the iono/atmosphere. However, for the solar particles penetrating at larger angles θ, formulas (53 - 56) are not exact. In the case of larger angles $\theta > 75^o$ the sphericity of the iono/atmosphere must be regarded similarly to the case of electromagnetic ionizing radiation, where the Chapman function Ch(θ, h) is introduced [10, 11]. In our case of particle ionization the following dependency is valid for solar cosmic rays:

$$(57)\qquad dE_k = \frac{5\times 10^2 \rho(h) Z^2 dl}{E_k}$$

where

$$(58)\qquad dl = dh \sec\theta$$

as $\rho(h)$ is the atmospheric density at the altitude h. If we integrate the ratio (57) and solve it in comparison with kinetic energy E_k we receive the law (53). Now the ionizing capability for **monoenergetic particles** will be:

$$(59)\qquad C(h) = 15 Z^2 \left(E_k^2 - 10^3 \tilde{h} Z^2 \sec\theta\right)^{-\frac{1}{2}}$$

in the case of flat model and

$$(60)\qquad C(h) = 15 Z^2 \left[E_k^2 - 10^3 \tilde{h} Z^2 Ch\left(\theta, R^*\right)\right]^{-\frac{1}{2}}$$

for the case of spherical model.

Equation (57) can be generalized if it is taken into the form:

(61) $$dE_{\mathrm{k}} = \frac{5\times 10^{2}\rho(h)Z^{2}dl}{E_{\mathrm{k}}^{\mathrm{m}}}$$

Than (59, 60) appears as a particular case of the common formula at m = 1. And at m = –1 the dependences of the electromagnetic ionized radiations are obtained [10, 11]. In the monographs [11, 22, 23] calculations were made for the ionization in the iono/atmosphere with parameter m = 3/4, which improved somewhat the model m = 1. In the contemporary models CORIMIA - *COsmic Ray Ionization Model for Ionosphere and Atmosphere* [12] and in the model of Dorman [23] at the analytical approximations are used different variable values in the range m = –0.1 ÷ 1 for the corresponding energy intervals. For the sub-relativistic particles (SCR and ACR) the values m of the ionization expression (61) are in the range m = 1 ÷ 0.1. By increasing of the energy of CR particles decreases the parameter m. At high energies ≈ 1 GeV/nucl the parameter m falls to 0. For the relativistic energies > 3-5 GeV/nucl (which are characteristic for GCR) begins slowly logarithmic increase of the ionization losses and the parameter m becomes m = –0.1.

Conclusion

In the present work we discovered a connection between the two approaches: the determination of ionizing capability *C* (used by the analytical model CORIMIA) and the ionization yield *Y* function (used by the statistical code CORSIKA). We found a consistent method for the calculation of atmospheric ionization due to cosmic rays with galactic, solar and interplanetary (anomalous CR) origin.

The results of the full Monte Carlo simulation which are tabulated in a form of the ionization yield function [1] can be applied much more widely. With the help of the achievements of this work the analytical models CORIMIA can use the results of statistical code CORSIKA that will allow comparison between the two approaches. As is known by CORSIKA is possible to estimate the CR ionization below 20-25 km and by the model

CORIMIA - above this altitude. So those two approaches are complementary.

So two useful approaches have been developed to compute the CR ionization, e.g.: a) the model CORIMIA is analytical [2, 4, 9, 12, 15], while b) CORSIKA code, including the FLUKA Monte Carlo package, is based on a Monte Carlo simulation of the atmospheric cascade [1, 3, 13, 14, 16]. Usually the analytical models use the ionizing capability $C(h)$ function, while the statistical simulations utilize the ionization yield $Y(h)$ function.

In part 1.1. *Analytical and Statistical Approaches in Ionization Models and Relationships between them* we have found a new relationship between the functions $C(h)$ and $Y(h)$ and we have implemented a unified approach to the calculation of iono/atmospheric ionization due to cosmic rays with galactic, solar and interplanetary origin.

Further a theoretical considerations were carried in relation to these issues in the part 2. *Relativistic Case – GCR*, and part *3. Sub-relativistic Case – SCR and ACR* for the solar and anomalous CR ionization shall be the basis for a quantitative investigation of the mechanisms for solar-terrestrial influences. The GCR, SCR and ACR are a major factor affecting the physical-chemical processes in the iono/atmosphere, including the electrical conductivities, electrical currents and fields [24] and the iono/atmospheric chemistry - i.e. the variations and planetary distribution of the ozone [17, 18]. All this has important applications for the space weather and space climate [25, 26]. These results will stimulate the quantitative study of the physical processes and physical mechanisms in the Earth's environment and in the Sun – Earth system.

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НОВ АНАЛИТИТИЧЕН ПОДХОД ЗА МОДЕЛИРАНЕ НА ЙОНИЗАЦИЯТА НА КОСМИЧЕСКИТЕ ЛЪЧИ В ОКОЛОПЛАНЕТНИТЕ ПРОСТРАНСТВА ПОСРЕСТВОМ ИЗПОЛЗВАНЕТО НА ПОРАЖДАЩИ ЙОНИЗАЦИЯТА ФУНКЦИИ

П. Велинов

Резюме

Космическите лъчи (CR) въздействат върху йонизацията и електрическите параметри в атмосферата, а също така и върху нейните химически процеси (образуване и разрушаване на озона в стратосферата). CR йонизират цялата средна и ниска атмосфера, т.е. страто-мезосферата и тропосферата. Така че космическите лъчи са ключов фактор, както за атмосферната химия, така и за космическото време и космическия климат в околоземното космическо пространство.

Съществуват основно два подхода за изчисляването на йонизацията на CR, например: 1) аналитичният модел CORIMIA - COsmic Ray Ionization Model for Ionosphere and Atmosphere, и 2) статистическият модел CORSIKA, включващ програмата FLUKA, която се базира на симулацията на атмосферните каскади по метода Монте Карло. Обикновено аналитичните модели използват функцията на йонизиращата

способност C, докато статистическите симулации използват пораждащата йонизация функция Y. В настоящата работа е намерена връзка между функциите C и Y и е предложен единен подход за изчисляване на атмосферната йонизация вследствие на космическите лъчи с галактичен (GCR), слънчев (SCR) и междупланетен (аномални CR, ACR) произход.

В работата са изведени формули за йонизиращата способност и пораждащата йонизация функция при общия случай, при релативистична апроксимация (GCR) и при суб-релативистичен случай (SCR и ACR). Входните параметри на предложения модел включват пълния състав на групите ядра на космическите лъчи, както следва: протони (p, $Z = 1$), алфа частици (α, $Z = 2$), и HEZ частици: леки (L, $3 \leq Z \leq 5$), средни (M, $6 \leq Z \leq 9$), тежки (H, $10 \leq Z \leq 19$), много тежки (VH, $Z \geq 20$) и свръх тежки (SH, $Z \geq 30$) групи ядра.

В работата са дискутирани някои практически приложения на получените резултати. Така например, резултатите от пълната симулация по Монте Карло, където са табулирани пораждащите йонизация функции в удобна форма, могат да се приложат много по-широко. С помощта на постиженията на настоящата работа аналитичният модел CORIMIA може да използва резултатите от статистическата програма CORSIKA, което ще даде възможност за сравнение между двата подхода. Както е известно, с помощта на CORSIKA може да се оценява йонизацията на космическите лъчи под 20-25 km, докато с модела CORIMIA – над тези височини. Така че тези два подхода се допълват взаимно.

Предложеният подход може да се използва за количествени разглеждания и анализ на слънчево-земните връзки и за проблемите на космическото време и космическия климат. Това е един теоретичен подход и поради това той може да се приложи също така и за изчисляване на йонизационните ефекти в планетните йоносфери и атмосфери.